Dr. Pinky Rani
Guest Faculty
Department of Economics
Maharaja Bahadur Ram Ranvijay Prasad Singh College
Veer Kunwar Singh University, Ara
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Topic- Application of Maxima and Minima in Economics

Application of Maxima and Minima in Economics

Maximization of Output

Assuming that labour is the only variable factor , we can write the production function of a firm as x = f(L), where x denotes total product of labour which will be denoted as TPL.

The Average product of labour is APL = TPL/L = f(L)/L.

The Marginal product of labour is MPL = dx/dL = f′(L).

The Necessary condition for maximum output is d(TPL)/dL = dx/dL = MPL = 0

Often we are interested in finding the level of employment of labour at which its average product is maximum.

For maxima of APL, we have

$$d(APL)/dL = [\, L.f'(L) - f(L)\,]/L^2 = 0$$

L.f′(L) = f(L) or f′(L) = f(L)/L or MPL = APL.

Thus the marginal product and average product of a factor are equal at the maxima of the later.

Example : The short run production function of a manufacturer is given as $x = 11L + 16L^2 - L^3$

(i) Find the average product function, APL, the marginal product function , MPL, and show that MPL = APL where APL is maximum.

(ii) Find the value of L for which output is maximum.

Solution:

(i) APL = x/L = 11 + 16L - L2 .   MPL = dx/dL = 11 + 32L + L2

We have d(APL)/dL = 16 - 2L =0.   Thus, APL is maximum at L = 8.

Since d2(APL)/dL2 = -2 < 0, the second order condition is satisfied.

The maximum APL = 11 + 16 8 - 82 = 75

Further, MPL when L = 8, is 11 + 32 8 - 3 82 = 75.

Thus, APL = MPL when APL is maximum

(ii) For maximum output:

dx/dL = 11 + 32L - 3L2 = 0

or (11 - L)(1 +3L) = 0 , therefore L =11. The other value being negative is dropped.

Since d2x/dL2 = 32 -6L = -34 < 0, the

second order condition for maxima is satisfied

Minimization of Cost

If total cost C = F(x) , then we can define AC = C/x = F(x)/x, and

MC = dC/dx = F′(x)

Very often we're interested in finding the level of output that gives minimum Ac. For minima of AC we have:

dAC/dx =[x.F′(x) - F(x)]/x2 = 0

x.F′(x) = F(x) or F′(x) = F(x)/x or MC = AC

Thus, marginal cost is equal to the average at the minima of the later.

Example: The cost of fuel consumed per hour in running a train is proportional to the square of its speed (in kms per hour), and it costs Rs 3200 per hour at a speed of 40 kms per hour. What is the most economical speed, if the fixed charges are 12,800 per hour ?

Solution: Let F be the cost of fuel and x be the speed of the train per hour.

We are given that F is directly related to x2 or F = kx2, where k is a constant of proportionality.

When x = 40, F is given to be 3200, therefore k = 3200/1600 = 2.

Thus we can write F = 2x2 , as the cost of fuel per hour of running the train when its speed is x kms per hour.Now the total cost of running the train when its speed is x kms per hour is TC = 12800 + 2x2

Average cost AC = 12800/x + 2x

The most economical speed will be that value of x which minimises AC.

AC/dx = -12800/x2 + 2 = 0, for minima

or x2 = 12800/2 = 6400 or x = 80 kms/hour.

Second order condition

d2(Ac)/dx2 = 25600/x3 > 0, when x = 80.

Thus, the second order condition for minima is satisfied.